Computational investigation of generalized intersection cuts
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INTRODUCTION

Input: MIP: \[ \min\{c^T x: A x \geq b, x \geq 0, x_j \in \mathbb{Z}, j \in I\} \]

Notation:
- \( P := \{x: A x \geq b, x \geq 0\} \)
- \( P_I := \{x \in P: x_j \in \mathbb{Z}, j \in I\} \)
- \( \bar{x} = \arg\min_{x \in \mathbb{R}^n} \{c^T x, x \in P\} \)

Goal: A non-recursive method to generate valid cuts

Motivation: Avoid numerical issues encountered in standard recursive cutting plane procedures

Idea: Activate hyperplanes to obtain a tighter relaxation of \( P_I \); full activation computationally expensive, hence partial hyperplane activation (PHA)

Partial activation

PHA-associated: Intersect each ray of \( C(\mathbb{R}^n) \) with a hyperplane, activating it (partially) on that ray alone

Valid cuts: Consider the system, for \( \beta \in \{-1,1\} \):

- \( \alpha x \leq \beta, \quad \beta' \in \mathbb{R} \)
- \( \alpha x \geq \beta, \quad \beta' \in \mathbb{R} \)

Here, \( \mathbb{P} \) and \( \mathbb{R} \) are rays and points generated by PHA-\( \alpha \). Any feasible solution \( \bar{x} \) with \( \beta = \beta' \) yields a valid cut \( x \leq \beta \) for \( P_I \).

Computational investigation: Experiment with various options for choosing hyperplanes in PHA-\( \alpha \), test effect of cutting rays by additional hyperplanes, and compare strength of cuts obtained from different objectives used with the cut LP

RESULTS

Experimental setup: Instances selected from MIPLIB 3 based on time taken to test one set of parameters. Compared generalized intersection cuts (GICs) to standard intersection cuts (SICs), which are known to be strong.

Hyperplane selection. Choose hyperplane that: 

- \( (1.1) \) Intersects ray first
- \( (1.2) \) Gives intersection points with best average depth
- \( (1.3) \) Creates largest number of final intersection points (final means the point is in \( P \))

Cut selection. 

- \( (1) \) Cut LP with objective
- \( (2) \) Solve a bilinear program: \( \min_{x} \{ \alpha x^T \beta; \beta' \in \mathbb{P} \} \)

Number of hyperplanes cutting a ray: Test effect of activating up to three hyperplanes per ray (+1H, +2H, +3H). First hyperplane selected by one of rules above; additional ones activated to maximize number of final intersection points.

In the (separable) bilinear program, \( \beta \) refers to \( P \) intersected with all the standard intersection cuts. It is solved iteratively over each of the variable sets, which only appear together in the objective.

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<th>( P )</th>
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Table 1: Percentage gap closed by hyperplane activation procedure

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CONCLUSION

Activating an additional hyperplane per split increases the strength of cuts. However, activating a third hyperplane sometimes leads to worse cuts.

Investigation showed that strategy \( C \) performs nearly as well as more sophisticated methods (S, B).

Future research will aim to address the questions:

1. Why are there so few GICs generated?

2. Why does the third hyperplane, while adding more deep and final points, lead to worse cuts?

3. How do we identify good objectives to use in the cut LP?

What is the effect of using other cut generating sets such as triangles and parametric octahedra?

REFERENCES